4.2 - Reduction of Order

Consider a 2nd-order DE

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0 \qquad (1)$$

The reduction of order formula is

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} dx$$

Example: The indicated function $y_1(x)$ is a solution of the given differential equation. Use reduction of order (the process or the formula) to find a second solution $y_2(x)$.

$y'' + 9y = 0; \quad y_1 = \sin 3x$

Example: $6y'' + y' - y = 0; \quad y_1 = e^{x/3}$

Example: $x^2y'' - 3xy' + 5y = 0; \quad y_1 = x^2\cos(\ln x)$

Example: The indicated function $y_1(x)$ is a solution of the associated homogeneous equation. Use the method of reduction of order [the process] to find a second solution $y_2(x)$ of the homogeneous equation and a particular solution $y_p(x)$ of the given nonhomogeneous equation.

$y'' - 4y' + 3y = x; \quad y_1 = e^x$